1 $\int a^u du$ $\frac{a^u}{\ln a} + c$

2 $\int \frac{1}{\sqrt{1-u^2}} du$ $\sin^{-1} u + c$

3 $\int \cot u \, du$ $-\ln|\csc u| + c$

4 $\int \sec u \tan u \, du$ $\sec u + c$

5 $\int \cos u \, du$ $\sin u + c$

Nov 24-8:56 AM

35. $y = 2e^{-t}\cos t$ $t \geq 0$

$0 \leq t \leq 2\pi$ $\bar{y} = \frac{1}{2\pi - 0}\int_0^{2\pi} 2e^{-t}\cos t \, dt = \frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t \, dt$

$\int e^{-t}\cos t \, dt = e^{-t}\sin t + \int e^{-t}\sin t \, dt$ parts

$u = e^{-t}$ $dv = \cos t \, dt$ | $u = e^{-t}$ $dv = \sin t \, dt$

$du = -e^{-t}dt$ $v = \sin t$ | $du = -e^{-t}dt$ $v = -\cos t$

$\int e^{-t}\cos t \, dt = e^{-t}\sin t + -e^{-t}\cos t - \int e^{-t}\cos t \, dt$

$\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t \, dt = \frac{e^{-t}\sin t - e^{-t}\cos t}{2\pi}\Big|_0^{2\pi}$

$= \frac{1}{2\pi}\left\{\left(e^{-2\pi}\sin 2\pi - e^{-2\pi}\cos 2\pi\right) - \left(e^{-0}\sin 0 - e^{-0}\cos 0\right)\right\}$

$= \frac{1}{2\pi}\left[\left(0 - e^{-2\pi}\right) - (0 - 1)\right] = \frac{-e^{-2\pi} + 1}{2\pi}$

Nov 24-9:20 AM

6.4a Exponential Growth and Decay

law of exponential change

If y changes at a rate proportional to the amount present

that is, if $\boxed{\frac{dy}{dt} = ky \text{ then } y = y_0 e^{kt}}$

diff. eq solution

$k > 0$ growth

$k < 0$ decay

$k =$ growth/decay rate

$\frac{dy}{dt} = ky$ seperate variables

$\int \frac{dy}{y} = \int k \, dt$ integrate

$e^{\ln|y|} = e^{(kt + c)}$ solve for y

$|y| = e^{kt} \cdot e^{c}$

$y = Ae^{kt}$

$y = y_0 e^{kt}$

$t = 0$

$y = y_0$

initial condition

$y_0 = Ae^{k0} = A$

Dec 2-1:38 PM

Suppose you deposit $800 in an account that pays 6.3% annual interest compounded continuously. How much do you have 8 years later?

$\frac{dy}{dt} = .063y$ $y = 800e^{.063t}$

$y(8) = 800e^{.063 \cdot 8} = 1324.26$

Find the doubling time. 11 yrs

doubling time if you start with 1000? 11

800:

$1600 = 800e^{.063t}$

$2 = e^{.063t}$

$\ln 2 = .063t$

$\frac{\ln 2}{.063} = t$

1000:

$2000 = 1000e^{.063t}$

$2 = e^{.063t}$

$\boxed{D = \frac{\ln 2}{k}}$

y_0

$2y_0 = y_0 e^{kt}$

$2 = e^{kt}$

$\ln 2 = kt$

$\frac{\ln 2}{k} = t$

Dec 2-1:45 PM

Radioactive decay and half-life

$$\frac{dy}{dt} = -ky \qquad y = y_0 e^{-kt}$$

Find the half-life

$$\frac{1}{2}y_0 = y_0 e^{-kt}$$
$$\frac{1}{2} = e^{-kt}$$
$$\ln\frac{1}{2} = -kt$$
$$\frac{\ln\frac{1}{2}}{-k} = t$$
$$\frac{\ln 1 - \ln 2}{-k} = t$$
$$\boxed{\frac{\ln 2}{k} = H}$$

Dec 2-1:47 PM

Choosing a convenient base

$$y = y_0 2^{\frac{t}{D}}$$ D=Doubling Time

$$y = y_0 (\frac{1}{2})^{\frac{t}{H}}$$ H = Half-life

$$y = y_0 \cdot 2^{\frac{t}{(\frac{\ln 2}{k})}}$$
$$= y_0\, 2^{\frac{k}{\ln 2}t}$$
$$= y_0 (2^{\frac{1}{\ln 2}})^{kt}$$
$$y = y_0 e^{kt}$$

Dec 2-1:53 PM

The half-life of carbon 14 is about 5700 years. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

$$y = y_0 e^{-kt}$$
$$.90 \cancel{y_0} = \cancel{y_0} e^{-.000122t}$$
$$\ln .90 = -.000122t$$
$$t = \frac{\ln .90}{-.000122} = 866 \text{ yrs}$$

$$H = 5700 = \frac{\ln 2}{k}$$
$$k = \frac{\ln 2}{5700}$$
$$k = .000122$$

10% decay, 90% remains

$$y = .90\, y_0$$

Dec 2-1:57 PM

Ex 4. At the beginning of the summer the population of a hive of wasps is growing at a rate proportional to the population. From a population of 10 on May 1, the number of hornets grows to 50 in 30 days. If the growth continues to follow the same model, how many days after May 1 will the population reach 100?

$t=0$, $y=10$ $\quad$ $t=30$, $y=50$ $\quad$ $t=?$, $y=100$ $\quad$ exp. growth

$$k = \frac{\ln 5}{30}$$
$$k = .053648$$

$$100 = 10 e^{.053648t}$$
$$10 = e^{.053648t}$$
$$\frac{\ln 10}{.053648} = \cancel{.053648}t$$
$$43 = t$$

Dec 2-2:00 PM

$t=0$ $t=30$ $t=?$

$y=10$ $y=50$ $y=100$

$y = 10 \cdot 5^{\frac{t}{30}}$

$100 = 10 \cdot 5^{\frac{t}{30}}$

$10 = 5^{t/30}$

$\log_5 10 = \frac{t}{30}$

$\ln 10 = \ln 5^{t/30}$

$t = 30 \log_5 10$

$\ln 10 = \frac{t}{30} \cdot \ln 5$

$t = 43$

$43 = \frac{30 \ln 10}{\ln 5} = t$

Nov 24-10:21 AM